Int. J. Heat Mass Transfer. Vol. 36, No. 6, pp. 1435–1444, 1993
Printed in Great Britain

0017-9310/93 $6.00 + 0.00

An extension of the fire-field modelling technique to include fire–sprinkler interaction—I. The mathematical basis

N. A. HOFFMANN and E. R. GALEA
Centre for Numerical Modelling and Process Analysis, The University of Greenwich, London, U.K.

(*Received* 17 *September* 1991 *and in final form* 28 *August* 1992)

Abstract—This paper considers an extension of the fire field modelling technique to include two-phase fire-sprinkler scenarios—namely the interaction between hot combustion gases and the sprinkler spray. The two techniques currently available to simulate a spray, namely the Lagrangian particle-tracking method and the Eulerian volume-fraction method, are discussed. The mathematical basis of a transient three dimensional computer model based on the latter type is described within this first part.

1. INTRODUCTION

THE DESTRUCTIVE powers of uncontrolled fires can be seen daily [1]. These result in around a thousand deaths and in excess of £600 million worth of property loss a year in the United Kingdom alone. Because of these grim statistics the mathematical modelling of fire and smoke spread in enclosures has received a considerable amount of attention. This has led to the development of two distinct modelling strategies: zone and field modelling [2]. The Harvard Fire Code [3] and JASMINE [4] are examples of these two methods.

Fire models are now able to provide us with a clearer understanding of the complex processes governing compartment fires. In comparison, the equally important area of fire suppression has not received as much attention and thus remains in a relatively primitive state of development.

While the mathematical modelling of fire suppression is a relatively recent activity, the idea of using suppressants such as water to control and quench fires is as old as time. The first recorded use of a purpose built fire suppression system to protect a building occurred in 1812, when the proprietors of the Theatre Royal Drury Lane, London installed a prototype sprinkler system [5]. Today, the use of automatic sprinkler systems is widely accepted as the most effective means of automatic fire protection [6]. Insurance statistics show that only four percent of reported losses occur on sprinklered premises [7]. In fact, sprinklers successfully control or extinguish fires in 98.5% of cases [8] and often extinguish fires before the arrival of the local fire brigade [9, 10].

In order to efficiently combat fire it is necessary to understand the nature of the interaction between the hot combustion products and the liquid water. Armed with this knowledge, fire engineers are able to optimise the design and location of sprinkler devices. Factors which need to be considered include water flow rate, spray pattern, droplet size and the number and location of sprinkler heads. Furthermore, issues such as whether or not the compartment is vented can influence the effectiveness of the sprinkler system.

The design, location and installation of these devices rely on physical experimentation and experience. The amount of human and financial resources required to carry out full-scale testing with completely fitted enclosures can be extremely expensive. Additionally, it is not always possible to conduct sufficient fire tests to adequately deal with all alternatives, such as the nature and position of fire sources, ventilation configurations and sprinkler options.

The mathematical modelling of fire-sprinkler systems provides a means to overcome these difficulties.

2. REVIEW OF FIRE-SPRINKLER MODELLING

The study of fire-sprinkler interaction can be divided into the investigation of two main physical phases and their interaction. These are the gas phase, involving the general fluid circulation of the hot combustion products within the compartment, and the liquid phase, representing the evaporating water droplets which have been injected into the fire compartment through the sprinkler orifice. As in the case for fire modelling, two distinct approaches have been developed to deal with various aspects of these interacting phases; the established method of 'back of envelope type calculations' which now have been largely computerised, and a more fundamental approach.

2.1. *Established models*

The processes governing the interaction between the two phases have been studied extensively. During a series of experiments Rasbash [11] studied the effect

NOMENCLATURE

A	total surface area in control cell [m^2]
A_p	total projected surface area [m^2]
b	mass transfer coefficient [$m\ s^{-1}$]
c	concentration of water vapour [$kg\ m^{-3}$]
C_D	drag coefficient
Cp	specific heat [$J\ kg^{-1}\ K^{-1}$]
d	droplet diameter [m]
D	diameter of fire source [m]
F	interphase friction force [N]
h	enthalpy [$J\ kg^{-1}$]
$\hat{h}$	heat transfer coefficient [$W\ m^{-2}\ K^{-1}$]
hum	relative air humidity [%]
k	kinematic turbulence kinetic energy [$m^2\ s^{-1}$]
l	flame length [m]
L	latent heat [$J\ kg^{-1}$]
$\dot{m}$	rate of mass transfer [$kg\ s^{-1}$]
M	molecular weight [$kg\ kmol^{-1}$]
p	pressure [atm]
$\dot{q}$	rate of heat transfer [$W\ s^{-1}$]
Q	total fire heat release [W]
r	volume fraction of phase considered
R_u	universal gas constant, $8.314 * 10^3$ [$J\ kmol^{-1}\ K^{-1}$]
S	source term
T	temperature [K]
V	velocity vector
V_{slip}	slip velocity [$m\ s^{-1}$]
Vol	volume of the cell [m^3].

Greek symbols

Γ	diffusion coefficient
ε	turbulence dissipation rate of k [$m^2\ s^{-1}$]
λ	thermal conductivity [$W\ m^{-1}\ K^{-1}$]
ν	laminar kinematic viscosity [$m^2\ s^{-1}$]
ρ	density [$kg\ m^{-3}$]
Φ	general dependent variable.

Subscripts

g	gas phase
i	refers to phase in question; g or l
l	liquid phase
s	surface
sat	saturation.

Dimensionless numbers

Nu	Nusselt number
Re	Reynolds number
Sc	Schmidt number
Sh	Sherwood number.

of water sprays on fires in order to be able to define the mechanism of extinction. Empirical relationships were determined from this study to evaluate heat transfer between the water sprays and flames. The effect of the interaction between sprinkler sprays and a smoke layer beneath a ceiling was studied by Bullen [12]. He explained why the effect of smoke logging occurred in certain situations with respect to the drag force to buoyancy ratio. However, the cooling effect of the water spray on the smoke layer, not considered by Bullen, was studied by Morgan [13]. He devised a model to describe the interaction between the spray and the smoke layer taking into account the heat removed from the layer and the drag force of the spray.

A considerable amount of research has also been invested into the characteristics of sprinkler systems, such as their thermal response and measured by the response time index [14]. The index is a product of the thermal time constant of the heat-responsive element of the sprinkler system and the square root of the associated gas velocity.

Based on these fundamental research studies PC based computer packages have been developed to aid the understanding of the development of the fire conditions. Evans and Stroup [15] produced the computer model DETACT-T2 capable of calculating the time required for a ceiling mounted heat and smoke detector to respond to a growing fire. A recently published package FPETOOL [16] is a further example of computerised engineering equations and models. It addresses problems related to the fire development and the resulting conditions within a building including means of calculating activation of sprinklers and detectors.

2.2. *Fundamental models*

A more thorough approach incorporates more fundamental relationships such as mass conservation and Newton's Law of motion. These are used to study aspects such as the temperature distribution within a compartment due to a fire, or even the movement of the droplets injected through a sprinkler nozzle. In this way some of the empiricism inherent in the more established models may be eliminated.

The simulation of fire-sprinkler interaction falls into the category of multi-phase processes. Two-phase, and multi-phase flow in general play an important role in a wide range of environmental, industrial and engineering disciplines. Typical examples are fluidisation, sedimentation, air and water pollution and spray drying and cooling.

Particulate gas–liquid, solid–liquid and solid–gas flows form one class of two-phase flows, and are usually referred to as dispersed flows. Fire-sprinkler interaction falls into this class of analysis.

A model developed by Gardiner [17], which he describes as a quasi-zone-field model is capable of

describing the interaction between sprinklers and the thermally buoyant layers of fire gases. It is a three-dimensional, steady-state model which takes into account the geometry of the compartment and the position and characteristics of the sprinkler head(s). However, some quite restrictive assumptions were made for the development of the model. Firstly, it was assumed that a fire is positioned within a long straight corridor. This enabled idealised buoyancy profiles to be fitted further along the corridor where the sprinklers were assumed to be located. The fire gases are assumed to be at steady-state, hence no account can be made of fire growth or suppression. From this basis the droplets are released into the compartment, their trajectories calculated and allowed to interact with the gases. The results are then able to show the complete physical and thermal histories of the spray including the number of drops which have evaporated or hit the boundaries. More importantly, the effect on the smoke layer can be seen as the spray passes through it.

The interaction between fire-induced air flow and water sprays can also be dealt with using two distinct approaches as extensions to the field fire model. One method treats the fluid phase as a continuum and the particulate second phase as individual particles. The momentum and transport of these discrete particles are calculated by taking into account the various forces they experience. The effect that the particles have on the gas phase is taken into account by introducing appropriate source terms in the gas phase conservation equations. This approach is known as the Lagrangian or particle tracking method. Alternatively, the two phases may be treated as two interspersed continua occupying the same space, their share of space being measured by their volume fractions [18]. It is then necessary to solve the appropriate continuity equations for the fluid and the particulate phases. This approach is known as the Volume Fraction or Eulerian method.

The treatment of the continuous phase is not discussed here as both methods deal with it in an identical manner common to all fire field models [19–21].

Any model simulating suspended droplets within a gas field needs to account for the effect of three distinct phenomena. As droplets are introduced into a gas field a drag force will be exerted on the gas and the droplets, accelerating or decelerating the overall flow accordingly. This momentum transfer between the phases is one of the coupling phenomena which needs to be considered. If the gas phase and the particles are initially at different temperatures, heat will be transferred between the two phases. As a result their densities can be altered affecting the overall flow field of the two phases. The last mode of coupling is mass transfer. This occurs if, as for the case of evaporating water particles, the mass of the second phase is decreasing. This change in mass must be accounted for by making appropriate adjustments to the gas phase.

2.3. *The Lagrangian method*

The first reported application of this technique is due to Migdal and Agosta [22]. They derived a system of differential equations from the basic laws of mass conservation and thermodynamics, to describe the dynamics of gas–particle systems in one-dimension. Within this system the gas phase was treated as a continua, while the particles, which were assumed to be finely dispersed, were treated as a continuous system of sources/sinks to the gas phase.

The Particle-Source-In-Cell (PSI-Cell) model of Crowe *et al.* [23] is a two-dimensional extension of the work of Migdal and Agosta. This model has successfully been applied to simulate a variety of processes including; spray drying, electrostatic precipitators, cyclone separators, combustors, and coal-fired furnaces. However, the technique is limited to low-turbulence, low-droplet-concentration situations [24–26]. This last restriction is primarily due to the significant computational effort required in calculating high-droplet-concentration flows.

With respect to fire-sprinkler modelling early notable work using particle tracking was carried out by Alpert [27–29]. These models utilised an extended version of the computer code TEACH-T [30]. The modified version could model gas droplet flows in two dimensions and took into account all three modes of gas droplet interaction. The droplet trajectories and size and temperature histories were obtained by integrating the equations of motion for each of the droplets within the gas phase using expressions for the interface mass and heat transfer rates. The droplet velocities, size and temperature along the trajectories were calculated using the Lagrangian approach.

This approach has also been adopted by Chow and Fong [31, 32]. They developed their own three dimensional transient field fire model in which they built into the capability of two-phase interaction using the PSI-Cell technique. However, as yet they considered only momentum and heat coupling between the phases as the effect of evaporation is much smaller than the convective part. The sprinkler discharge pattern was simulated as a constant hollow cone which was assumed to be unaffected by the air flow and hence no droplet trajectories were calculated. However, the effect of drag on the air flow was accounted for within the respective source terms. The model was applied to a corridor type enclosure fitted with 14 sprinkler heads, three of which were active during the simulation. The numerical grid fitted to the enclosure contained 8721 computational cells and required 20 hours of CPU time on a VAX 6420. The results obtained indicate how the water spray interacts with the hot gaseous smoke layer. The temperatures were also reduced whereby the water curtain was able to confine the fire.

2.4. *The Eulerian method*

The Eulerian Volume Fraction two-phase approach is a natural extension to the standard single phase

Eulerian method, whereby account has to be made of the volume that each phase occupies within a single control-cell.

For the single-phase Eulerian approach, the complete volume or space of each control-cell consists of the fluid under consideration; gas or liquid. In the case of the two-phase approach the volume of a single control-cell is made up of a mixture of gas and liquid, gas and solid, or liquid and solid, depending on the process simulated. In this way, assuming that the cell does not consist of blockages, a simple space sharing equation can be used to take into account the volume fraction of the cell consisting of the various phases. If a control-cell consists of only a single phase, the set of partial differential equations for the two phases reverts to the conventional single-phase system.

This approach has been embedded into the workings of PHOENICS [33, 34], a computational fluid dynamic code capable of modelling steady and transient, turbulent, three-dimensional two-phase problems.

The two-phase capabilities of PHOENICS have been demonstrated in a number of applications. Gosman *et al.* [35] simulated steady-state recirculating flow assuming no momentum interaction. Kostamis [36] performed a three-dimensional steady-state study of the flow characteristics in the off-gas ducting system of a steel making plant. In this study he included the effects of turbulence on the gas phase, chemical reactions and two-phase thermal radiation. Fenech [37] simulated an iron blast furnace using a two-dimensional steady-state model taking into account heat transfer and momentum coupling.

The two phases are treated as two interspersed continua occupying the same space, their share of space being measured by their volume fraction. The two phases interact with each other via friction, heat and mass transfer, and the calculations are dependent on the amount of each phase within each control volume.

In order to simulate processes which involve mass transfer between the two phases, modification to the standard procedure is required. Known as the shadow volume fraction method [38] this technique evaluates a further volume fraction as if no mass transfer had occurred. With the help of this and the liquid volume fraction—which takes into account mass transfer—evaluation of the changing particle diameters can be carried out. This method which has been imbedded into the PHOENICS code has already been used in steady-state simulations of the spray cooling of combustion products [39] and the ignition of granulated propellants within a gun barrel [40].

Of more direct interest is the application of the technique to fire-sprinkler systems. The technique has been applied to the steady-state [20, 41] and transient [20, 42, 43] three-dimensional analysis of fire-sprinkler interaction. The model, incorporating the three modes of gas-droplet interaction, predicts gas phase flow fields and temperature distributions prior and subsequent to sprinkler activation. Liquid water concentrations as well as velocity and temperature distributions for the liquid phase were also produced. In these studies the fire is seen simply as a heat source without the added complication of combustion.

In the remainder of this paper details of the above method will be outlined. The model considers the interaction between a sprinkler spray and combustion gases. The interaction between the flames and the spray and the subsequent cooling or even extinguishment of the fire are not considered at this stage. In addition, as the fire is treated as a volumetric heat source the effect of the water spray on unburnt fuel is not considered. Applications of the model to two distinct fire-sprinkler scenarios can be found in detail within a later publication.

3. THE MATHEMATICAL PROBLEM

The model consists of a set of non-linear partial differential equations describing the flow, heat and mass transfer, and the volume fractions of two interacting phases. The heat source, representing the fire, creates a strong buoyancy driven flow with large scale turbulent motion which controls the diffusion of mass and momentum and mixing of the two phases. The non-uniform buoyancy forces not only drive this flow but also increase turbulent mixing in the rising plume and inhibit it in hot stratified layers. The droplets that are injected into the above field are treated as a second continuous phase, interacting with the gaseous field. Once these two phases start interacting, equations need to be solved which not only deal with the general conservation equations of mass, momentum and energy of each phase, but also take into account the important interphase processes between them.

The starting point of the analysis is the set of partial differential equations that govern the phenomena under consideration. This set consists of the following equations: the continuity equation of each phase; the momentum equations that govern the conservation of momentum per unit mass for each phase, in each of the space directions (the Navier–Stokes equations, of which there are six); the equations for conservation of energy for each phase; species concentration (water vapour); and the equations for a turbulence model—in this case the k–ε model with standard constants [44]—for the gaseous phase only. In total there are thirteen equations which define the basic model compared with seven in the standard volumetric heat source field model. Considerably more computer power is required to solve this enhanced set of equations. In addition, SIMPLEST [18], the standard numerical procedure used to satisfy continuity and solve the equations, must be modified.

3.1. *The dependent and independent variables*

The independent variables used are the width, height and length; x, y and z of a cartesian co-ordinate system.

The 15 dependent variables requiring solution are

the six velocity components (u_1, v_1, w_1) for the gas phase and (u_2, v_2, w_2) for the particulate phase in their respective cartesian direction (x, y, z), along with the pressure p, which is assumed to be the same for both phases. The enthalpies for the gas and water phases, h_g and h_l respectively, along with the concentration of water vapour within the gas phase, c. The gas and liquid volume fractions r_g and r_l are solved for including the effect of evaporation. The 'shadow' volume fraction, r_s, is the volume fraction in the absence of evaporation. Finally the turbulence kinetic energy and dissipation rate of the gaseous phase (k, ε). Turbulence in the liquid phase is neglected. The 'shadow' volume fraction technique allows us to evaluate the diminishing droplet size during evaporation [38].

3.2. *The governing differential equations*

The transfer of heat and mass is governed by the conservation principles of mass, momentum, energy, etc. These principles can be expressed in terms of differential equations which possess a common form. The form of the generalised conservation equation is:

$$\underset{\text{transient}}{\frac{\partial}{\partial t}(r_i\rho_i\Phi_i)} + \underset{\text{convection}}{\text{div}\,(r_i\rho_i V_i\Phi_i)} - \underset{\text{diffusion}}{r_i\Gamma_\Phi \text{ grad } \Phi)} = \underset{\text{source}}{r_i S_{\Phi i}} \quad (1)$$

where Φ represents a general fluid property such as velocity or enthalpy. ρ represents the density of the phase, V is the velocity vector of the three velocity components. Γ_Φ is the diffusion coefficient which represents the general fluid properties such as viscosity or conductivity. The source term S_Φ deals with the general forces acting on the fluid such as heat generation or heat losses. r is the volume fraction and the subscript i refers to the phase in question; gas (g) or liquid (l).

3.3. *The conservation of mass equations*

The pressure variable is associated with the continuity equations:

$$\frac{\partial}{\partial t}(r_i\rho_i) + \text{div}\,(r_i\rho_i V_i) = r_i S_i \quad (2)$$

where the volume fractions are related to each other by the *space sharing* condition:

$$r_g + r_l = 1.0. \quad (3)$$

The source/sink terms S_i take into account the mass transferred between the two phases due to the evaporation of the particles.

3.4. *The conservation of momentum equations*

The conservation equation of momentum for the gas and liquid phases is given by:

$$\frac{\partial}{\partial t}(r_i\rho_i) + \text{div}\,(r_i\rho_i V_i\Phi_i - r_i\Gamma_\Phi \text{ grad } \Phi) = S_\Phi \quad (4)$$

where Φ stands for u_g, u_l, v_g, v_l, w_g and w_l. The source terms for S_Φ are given in Table 1.

3.5. *The conservation of energy equations*

The conservation of energy equation for the gas phase is given by:

$$\frac{\partial}{\partial t}(r_i\rho_i) + \text{div}\,(r_i\rho_i V_i h_i - r_i(\lambda_i/Cp_i) \text{ grad } h_i) = S_i. \quad (5)$$

The term on the right hand side is the source term which prescribes the rate of heat transferred from one phase to another.

3.6. *The auxiliary relations*

Due to the nature of the problem, i.e. the interaction between two phases, certain correlations need to be included in the model to close the problem. These time-dependent relations deal with the interphase heat and mass transfer and the friction between the gas and the liquid phases. The assumptions made are that the gas and droplets are dispersed within the control-volume and that the droplets are spherical. The last assumption is not essential but simplifies the nature of the empirical input [20].

3.6.1. *The interphase-friction coefficient.* The ability to predict the interphase drag or the relative velocity between phases is of considerable importance for modelling a two-phase system, as the use of a reliable interphase drag correlation can significantly affect the results.

The interphase drag force experienced by the two phases is related to the particle Reynolds number by a drag coefficient C_D. The interface friction force for flow around a sphere is given by:

$$F = 0.5 * C_D * \rho_g * A_p * V_{slip}^2 \quad (6)$$

where ρ_g is the density of the gas phase, A_p is the total projected area of the liquid phase present within a control-volume given by the equation:

$$A_p = (1.5 * r_l * Vol)/d \quad (7)$$

where Vol is the volume of the cell and d the droplet diameter.

V_{slip} is the slip velocity between the phases calculated by the function:

$$V_{slip} = \sqrt{\{(u_g - u_l)^2 + (v_g - v_l)^2 + (w_g + w_l)^2\}}. \quad (8)$$

In a study by Clift and Gauvin [45] they show that the drag experienced by a solid particle moving through a fluid is dependent on a number of factors such as turbulence and particle shape. Their equation which takes these factors into account and is applicable for $Re < 10^5$ is given by:

$$C_D = \frac{24}{Re} * (1 + 0.15 * Re^{0.687}) + \frac{0.42}{1.0 + 4.25 * 10^4 * Re^{-1.16}} \quad (9)$$

Table 1. The diffusion coefficients and source terms for the two-phase conservation equations

Φ	Γ_Φ	S_Φ
1	0	0 (continuity)
u_g	μ_{eff}	$-\frac{\partial p}{\partial x}r_g+\frac{\partial}{\partial x}\left(\mu_{eff}\frac{\partial u}{\partial x}\right)+\frac{\partial}{\partial y}\left(\mu_{eff}\frac{\partial v}{\partial x}\right)+\frac{\partial}{\partial z}\left(\mu_{eff}\frac{\partial w}{\partial x}\right)+F(u_l-u_g)+\dot{m}u_l$
u_l	0	$-\frac{\partial p}{\partial x}r_l-F(u_l-u_g)-\dot{m}u_l+$source of particle momentum
v_g	μ_{eff}	$-\frac{\partial p}{\partial y}r_g+\frac{\partial}{\partial x}\left(\mu_{eff}\frac{\partial u}{\partial y}\right)+\frac{\partial}{\partial y}\left(\mu_{eff}\frac{\partial v}{\partial y}\right)+\frac{\partial}{\partial z}\left(\mu_{eff}\frac{\partial w}{\partial y}\right)-g(\rho-\rho_{ref})+F(v_l-v_g)+\dot{m}v_l$
v_l	0	$-\frac{\partial p}{\partial y}r_l-F(v_l-v_g)-\dot{m}v_l+$source of particle momentum
w_g	μ_{eff}	$-\frac{\partial p}{\partial z}+\frac{\partial}{\partial x}\left(\mu_{eff}\frac{\partial u}{\partial z}\right)+\frac{\partial}{\partial y}\left(\mu_{eff}\frac{\partial v}{\partial z}\right)+\frac{\partial}{\partial z}\left(\mu_{eff}\frac{\partial w}{\partial z}\right)+F(u_l-u_g)+\dot{m}w_l$
w_l	0	$-\frac{\partial p}{\partial z}r_l-F(w_l-w_g)-\dot{m}w_l+$source of particle momentum
H_g	$\mu_{lam}/Pr_{lam}+\mu_t/Pr_t$	$\dot{q}+[(Cint_g+\max(0,\dot{m}))*(H_{sg}-H_g)]$
H_l	0	$[(Cint_l+\max(0,-\dot{m}))*(H_{sl}-H_l)]$
k	μ_{eff}/Pr_k	$G_K-\rho\varepsilon+G_B$
ε	μ_{eff}/Pr_ε	$(\varepsilon/k)[(G_K+G_B)C_1-C_2\rho\varepsilon]$

where

$$G_K=\mu_t\{2[(\partial u/\partial x)^2+(\partial v/\partial y)^2+(\partial w/\partial z)^2]+[(\partial u/\partial z)+(\partial w/\partial x)]^2+[(\partial w/\partial y)+(\partial v/\partial z)]^2+[(\partial u/\partial y)+(\partial v/\partial x)]^2\}$$

$$G_B=\frac{\mu_t g}{\rho}\frac{\partial p}{\partial y}$$

where Re is the particle Reynolds number given by:

$$Re=d*V_{slip}/v \tag{10}$$

where v is the laminar kinematic viscosity.

Mass flux from the surface can reduce the drag coefficient [20, 46]. In the case of burning fuel droplets this is a significant phenomena. For small evaporating water droplets the mass efflux has little effect on the drag [47].

In the case where the particles enter the flame the drag coefficient is re-calculated using [11]:

$$C_D=18.5*Re^{-0.6}. \tag{11}$$

In order to know exactly when a droplet enters the flame, the length of the flame, l, needs to be known. This length is calculated from the relationship for enclosure fires [48]:

$$l=0.23*(Q/1000)^{0.4}-1.02*D \tag{12}$$

where Q is the total heat release of the fire and D is the diameter of the fire source.

3.6.2. *The interphase-heat transfer coefficient.* Heat is exchanged between the hot gases and the cold water droplets. Central to the following treatment is the concept of an interface between two phases, with temperature T_s. Then, using this third temperature, the rate of heat transferred from the gas to the interface, $\dot{q}_g$, and from the interface to the particle interior, $\dot{q}_l$ is evaluated using the relations:

$$\dot{q}_g=\hat{h}_g*A(T_g-T_s) \tag{13}$$

$$\dot{q}_l=\hat{h}_l*A(T_s-T_l) \tag{14}$$

where $\hat{h}_g$ and $\hat{h}_l$ are the heat transfer coefficients between the gas and the interface, and the interface and the liquid respectively. The temperatures T_g and T_l are given by:

For the gas phase:

$$T_g=h_g/Cp_g \tag{15}$$

For the liquid phase:

$$T_l=h_l/Cp_l \tag{16}$$

where Cp_g and Cp_l are the specific heats of the gas and liquid phases respectively.

The interphase or particle surface temperature (T_s) is obtained from an energy balance over a control-volume enclosing the interface [36, 40]. It is assumed that mass transfer occurs with respect to the surface temperature. Taking into account equations (13) and

(14) it can be shown that T_s is given by:

$$T_s = \frac{\hat{h}_g T_g + \hat{h}_l T_l}{\hat{h}_g + \hat{h}_l + \dot{m}(Cp_g - Cp_l)}. \quad (17)$$

It may be noted here, that the above heat transfer relations, (13) and (14), are based only on convection. Previous studies [11, 49] to evaluate the importance of radiation on particles, concluded that for droplets of about one millimetre in diameter, the magnitude of radiation absorbed is negligibly small.

When mass transfer takes place the latent heat of evaporation is accounted for in the mass and heat transfer balances.

The heat transfer coefficient from the gas to the interface, a_g is:

$$\hat{h}_g = Nu * \lambda_g / d \quad (18)$$

where Nu is the Nusselt number. Due to the relative motion of the fluid, the local heat transfer coefficient depends on the velocity and the temperature profile. From experimental investigations a number of correlations have been obtained for different flow conditions and geometries of the objects suspended within the fluid. The correlation used within this model to calculate the heat transfer between the gas and the evaporating water droplets is [50]:

$$Nu = 2.0 + 0.6 Re^{1/2} Pr^{1/3}. \quad (19)$$

The heat transfer coefficient, a_l is evaluated assuming a cubic temperature distribution within the particle given by [40]:

$$\hat{h}_l = \frac{3\lambda_l}{d/2} \quad (20)$$

where λ_l is the thermal conductivity of water.

3.6.3. *The interphase-mass transfer coefficient.* Mass transfer involves the transportation of a mixture from a region of higher to an area of lower concentration. If this transport is due to the motion of a fluid it is said to be convected. In this study, the rate of mass transfer, the evaporation of small water droplets within a hot gas atmosphere, is due to the concentration difference between the hot dry air produced by the fire and the saturated concentration level of the droplets. The equation governing the rate of mass transfer is analogous to the interphase heat transfer coefficients discussed in the previous section. Hence, the interphase mass transfer equation is given by:

$$\dot{m} = b * A(c_i - c_g) \quad (21)$$

where b is the mass transfer coefficient and c_i and c_g are the concentration of water vapour at the interface and in the gas respectively [51]. The mass transfer coefficient, b, is evaluated using:

$$b = (Sh * D)/d \quad (22)$$

where D is the mass diffusivity of water in gas, and d is the average droplet diameter. Sh is the Sherwood number which is analogous to the Nusselt number used within the heat transfer calculations.

As in the case of heat transfer, the evaporation from a spherical droplet into still air is given by $Sh = 2.0$. In the more general cases of forced convection, the correlation obtained by Ranz and Marshall [52] for evaporating water droplets was implemented:

$$Sh = 2.0 + 0.6 Re^{1/2} Sc^{1/3}. \quad (23)$$

Their study, which was initially restricted to $Re < 200.0$, water droplet diameter range between $0.6 * 10^{-4}$ to $1.1 * 10^{-3}$ m and air temperatures up to 220°C. This is adequate for the purposes of this model which is primarily concerned with air–spray interaction remote from the actual fire. However, they also showed that their results could be *extrapolated with remarkable accuracy five times* beyond their experimental range of Reynolds numbers.

The concentration of water vapour present within the air was calculated using the following equation [51]:

$$c_g = \frac{M_w * hum * p_{sat}}{R_u * T_g} \quad (24)$$

where M_w is the molecular weight of water, hum is the relative humidity of the gas, p_{sat} is the saturation pressure and R_u is the universal gas constant. The concentration of water present at the interface was calculated using [51]:

$$c_s = \frac{M_w * p_{sat}}{R_u * T_l}. \quad (25)$$

3.6.4. *Particle size calculation.* During the development of this model, several assumptions concerning the particles were made. The first was to assume a uniform droplet size with an initial diameter of 1 mm. This, however, is not a limitation of the approach as a method known as MIPSA (Modified IPSA) has been developed and implemented within the software package CASCADE [53]. This method is capable of handling multi-sized particles within a system.

A further assumption was that the particles were spherical. In fact water droplets which are smaller than a critical size are stable and do not break up. In order for the drops to retain their spherical shape when falling within a gas certain conditions need to be satisfied. Equations exist [54, 55] to evaluate the critical droplet diameter taking into account various flow conditions such as surface tension, density difference between the droplet and the gas and the slip velocity between the gas and the droplet.

The last assumption is concerned with the interaction of the particles themselves. During this study it was assumed that particles are sufficiently distant so as not to be able to interact. In that way it was assumed that they were not able to hit or coalesce with each other, hence diminishing or enlarging their diameter. Furthermore, bouncing off boundaries or collecting on walls or ceiling were not taken into con-

sideration at this point. However, water was able to collect on the floor.

For this model the distribution of the average particle size throughout the domain and the number of particles present within a control-volume was evaluated. The *shadow* volume fraction method developed by Spalding [38] specifically for finite-difference calculations was employed.

From the shadow volume calculations, the average diameter of the particles within each control-volume can be determined using the relation:

$$d = d_0(r_1/r_s)^{1/3} \tag{26}$$

where d_0 is the initial average diameter of the particles, which are assumed uniform across the sprinkler orifice.

3.6.5. *Boundary conditions.* There are basically two types of boundaries: solid and free. On solid boundaries the non-slip condition is employed for the gas phase. This condition is also imposed on the particulate phase as no method exists to easily incorporate such phenomena as bouncing on the boundaries within the Eulerian method. For the enthalpy equation a fixed temperature is assumed at the outside surface of the walls enclosing the compartment. The momentum flux to the walls obeys the wall function relationship of Launder and Spalding [44]. A similar approach for the flux of heat to the walls has indicated that satisfactory solutions may be obtained by lumping the heat loss effects together in a local empirical transfer coefficient [3]. For the kinetic energy of turbulence (k), a zero diffusive flux at the wall is used. The dissipation rate (ε) is calculated from the length scale proportional to the distance from the wall.

Flow domains are extended outside the fire compartments in the vicinity of open doorways [2, 56]. This is necessary in order to obtain the correct flow behaviour through the doorway. On the resulting free boundaries a fixed pressure boundary condition is imposed in order that the flow may enter or leave the domain depending on the internal pressure field.

3.7. *The numerical solution procedure*

The resulting control-volume equations form a set of simultaneous equations which are solved numerically using the iterative procedures SIMPLEST (SIMPLE ShorTened) and IPSA (Inter-Phase Slip Algorithm) [18, 57]. SIMPLEST is an improved version of the SIMPLE algorithm which is well documented [19]. The latter, a more elaborate solution procedure which is used for multi-phase flows, is able to handle the presence of two simultaneously present phases sharing a common pressure. It evaluates the increased number of governing equations of the flow and the strong interaction between them, such as interphase friction and mass, as well as the space sharing of the volume fraction condition.

These algorithms are built into the computer program PHOENICS [33, 34] which also uses the conventional staggered grid approach for solving finite-volume equations [20, 23]. During the simulations, converged solutions are obtained by iterating on the solution domain until the residuals of the variables solved for are below a certain tolerance value, typically 10^{-4}.

4. CONCLUSIONS

Over the past 15 years considerable effort has been invested in the development of detailed fundamental models for the simulation of enclosure fires. For the majority of this period, the area of fire-sprinkler simulation has more or less been ignored by the field modelling community. This has been due to the extreme complication involved in simulating the processes involved in transient, buoyant, turbulent, three-dimensional, two-phase phenomena. However, the development of numerical procedures, such as the particle tracking and volume fraction methods heralds the promise of progress.

The work presented above, which utilises the volume fraction method, outlines the first step in the development of a three-dimensional, transient Eulerian–Eulerian model to describe fire-sprinkler interaction. The model, while not attempting to simulate the extinguishment process, describes the interaction of water droplets with the hot turbulent atmosphere of the fire compartment. The three basic mechanisms of interaction, namely, momentum, mass and heat transfer are taken into consideration. While this work has considered the interaction between sprinkler sprays and combustion gases, the equally important areas of sprinkler spray–flame and sprinkler spray–fuel burning rate are the subject for further research.

The application of the model to investigate two experimental fire-sprinkler scenarios will be presented in a later paper.

Acknowledgements—The authors would like to thank Professor N. Markatos for his involvement in the early phases of this project, CHAM for allowing the use of PHOENICS, as well as the SERC and Arups for funding.

REFERENCES

1. Anon, £175 million fire claims in the first quarter of 1989, *Fire and Security Protection*, pp. 12–16, Aug. 1989.
2. E. R. Galea, On the field modelling approach to the simulation of enclosure fires, *J. Fire Prot. Engng* **1**(1), 11–22 (1989).
3. H. E. Mitler, The Physical Basis for the Harvard Computer Code, Home Fire Protection Tech. Report No. 34, Harvard Univ. (1978).
4. G. Cox, S. Kumar and N. C. Markatos, Some field model validation studies, *Proc. of 1st Int. Symp. on Fire Safety Science*, Gaithersburg, U.S.A., Oct. 1988.
5. P. Nash, The history of sprinklers, *Fire Surveyor* **2**(2) (1973).
6. Anon, Automatic sprinkler systems-components, FPA Bulletin, No. 34.
7. Anon, Advances in sprinkler technology, *Fire Prevention* **171**, 38 (1984).

8. Anon, American survey shows high success rate for sprinklers, *Fire Prevention* **170,** 30–33 (1984).
9. T. Z. Harmathy, On the economics of Mandatory Sprinklered Dwellings, *Fire Tech.*, pp. 245–261, Aug. 1988.
10. Anon, Fires controlled by sprinklers, *Fire Prevention* **226,** 48–49 (1990).
11. D. J. Rasbash, Heat transfer between water sprays and flames of freely burning fires, *Proc. Symp. Interaction between fluids and particles*, London, U.K. (1962).
12. M. L. Bullen, The effect of a sprinkler on the stability of a smoke layer beneath a ceiling, *Fire Research Note. Nr. 1016*, July 1974.
13. H. P. Morgan, Heat transfer from a buoyant smoke layer beneath a ceiling to a sprinkler spray : a tentative theory, *Fire Research Note. Nr. 1069*, July 1977.
14. G. Heskestad and G. B. Robert, Modelling of thermal responsiveness of automatic sprinklers, *Proc. of 2nd Int. Symp. Fire Safety Science*, pp. 603–612 (1989).
15. D. D. Evans and D. W. Stroup, Methods to calculate the response time of heat and smoke detectors installed below large unobstructed ceilings, *Fire Tech.* **22** (1986).
16. H. E. Nelson, FPETOOL-Fire protection tools for hazard estimation—An overview of features, *Proc. of Interflam '90 ;* Interscience Comm. Ltd., pp. 85–92 (1990).
17. A. J. Gardiner, The mathematical modelling of the interaction between sprinklers and the thermally buoyant layers of gases from fires, PhD Thesis, South Bank Polytechnic, London, U.K. (1988).
18. D. B. Spalding, Numerical computations of multiphase fluid flow and heat transfer, *Recent Advances in Numerical Methods in Fluids* (Edited by C. Taylor and K. Morgan), pp. 139–167. Pineridge Press (1980).
19. S. V. Patankar, *Numerical Heat Transfer and Fluid Flow.* McGraw-Hill (1980).
20. N. A. Hoffmann, Computer simulation of fire–sprinkler interaction, Ph.D. Thesis, Thames Polytechnic, London (1990).
21. N. C. Markatos, K. A. Pericleous and G. Cox, A novel approach to the field modelling of fires, *PhysicoChemical Hydrodynamics* **7,** 125–143 (1986).
22. D. Midgal and C. V. Agosta, A source flow model for continuum gas–particle flow, *J. Appl. Mech. Trans of ASME* **35**(4), 860–865 (1967).
23. C. T. Crowe, M. P. Sharama and D. E. Stock, The Particle-Source-in-Cell (PSI-Cell) Model for Gas-Droplet Flows, ASME ; 75-WA/HT-25 (1975).
24. C. T. Crowe, Review—Numerical methods for dilute gas–particle flows, *J. Fluid Engng* **104,** 297–303 (1982).
25. F. Durst, D. Milojevic and B. Schonung, Eulerian and Lagrangian predications of particulate two-phase flows ; A numerical study, *Appl. Math. Modelling* **8,** 101–115 (1984).
26. W. A. Sirignano, The formulation of spray combustion models ; Resolution compared to droplet spacing, *J. Heat Transfer* **108,** 633–639 (1986).
27. R. L. Alpert and M. K. Mathews, Calculation of large-scale flow fields induced by droplet sprays, FMR Report, FMRC J.I.OEOJ4.BU, Dec. (1979).
28. R. L. Alpert, Calculated spray water-droplet flows in a fire environment, FMR Report, FMRC J.I. OJOJ1.BU (1986).
29. R. L. Alpert, Numerical modelling of the interaction between automatic sprinkler sprays and fire plumes, *Fire Safety J.* **9,** 157–163 (1985).
30. A. D. Gosman and W. M. Pun, Calculation of recirculating flows, Lecture Notes Imperial College of Science and Technology, Dec. (1973).
31. W. K. Chow and N. K. Fong, Numerical studies of the sprinkler fire interaction using field modelling technique, *Proc. Interflam '90*, pp. 25–34, Univ. of Kent, Canterbury, 3–6 Sept. 1990.
32. W. K. Chow and N. K. Fong, Numerical simulation on cooling of the fire-induced air flow by sprinkler water sprays, *Fire Safety J.* **17,** 263–290 (1991).
33. D. B. Spalding, A general-purpose computer program for multi-dimensional one-two-phase flow, Prepr. 81-6 ; *Mathematics and Computers in Simulation.* North-Holland (IMACS) ; **23,** 267–276 (1981).
34. H. Rosten and D. B. Spalding, PHOENICS Beginners Guide and User Manual, CHAM TR/100 (1986).
35. A. D. Gosman, K. H. Li and D. S. A. Samaraweera, A numerical calculation procedure for two-phase recirculating flows, *5th Int. Conf. on Numerical Methods in Fluid Dynamics*, 212–219 (1976).
36. P. Kostamis, Computer modelling and analysis of particulate laden gas flows, Ph.D. Thesis, Thames Polytechnic, London, U.K. (1987).
37. K. A. Fenech, Analysing aspects of the performance of an iron blast furnace, Ph.D. Thesis, Thames Polytechnic, London, U.K. (1987).
38. D. B. Spalding, The *shadow* method of particle-size calculation in two-phase combustion, *19th Symp. on Combustion*, pp. 941–951. The Combustion Inst. (1982).
39. D. Kircaldy and N. C. Markatos, Spray cooling of combustion products, CHAM TR 1371/1 London (1982).
40. N. C. Markatos and D. Kircaldy, Analysis and computation of three-dimensional, transient flow and combustion through granulated propellants, *Fluids Eng. Division of the ASME*, 82-Fe-9 (1982).
41. N. A. Hoffmann, E. Galea and N. C. Markatos, Mathematical modelling of fire sprinkler systems, *Appl. Math. Modelling* **13,** 298–306 (1989).
42. N. A. Hoffmann and E. R. Galea, The mathematical modelling of two-phase fire–sprinkler interaction, *5th European Conf. on Mathematics in Industry ; Finland 1990*, pp. 249–253. B. G. Teubner, Stuttgart and Kluwer Academic Publishers (1990).
43. N. A. Hoffmann and E. Galea, An application of the Eulerian–Eulerian technique to a transient two-phase fire-sprinkler simulation, *13th IMACS World Congress, Dublin, Ireland* **4,** 1936–1938 (1991).
44. B. E. Launder and D. B. Spalding, *Mathematical Models of Turbulence.* Academic Press, London (1972).
45. R. Clift and W. H. Gauvin, Motion of entrained particles in gas streams, *Canadian J. Chem. Engng* **49,** 439–448 (1971).
46. H. Wise and G. A. Agoston, Burning of a liquid droplet, *Literature of the Combustion of Petroleum* **20,** 116–135 (1958).
47. M. C. Yuen and L. W. Chen, On drag of evaporating liquid droplets, *Combustion Science and Technology* **14,** 147–154 (1976).
48. D. Drysdale, *An Introduction to Fire Dynamics.* Wiley, Chichester (1985).
49. T. W. Hoffmann and W. H. Gauvin, Analysis of the radiative heat absorption in the boundary layer surrounding an evaporating drop, *Canadian J. Chem. Engng* **39,** 252–259 (1962).
50. W. E. Ranz and W. R. Marshall, Jr., Evaporation from drops : Part I, *Chemical Eng. Progress* **48**(3), 141–146 (1952).
51. F. Kreith and W. Z. Black, *Basic Heat Transfer.* Harper and Row, New York (1980).
52. W. E. Ranz and W. R. Marshall, Jr., Evaporation from drops : Part II, *Chemical Eng. Progress* **48**(4), 173–180 (1952).
53. M. K. Patel and M. Cross, The modelling of fluidised beds for ore reduction. In *Numerical Methods in Laminar and Turbulent Flows* (Edited by Taylor *et al.*), *Proc. of 6th Int. Conf.*, Swansea, U.K., Vol. 6(2) (1989).
54. F. H. Garner and J. J. Lane, Mass transfer to drops of liquid suspended in a gas stream, Part II : Experimental work and results, *Trans. Inst. Chem. Engng* **37,** 162–172 (1959).
55. G. D. Gordon, Mechanism and speed of breakup of drops, *J. Appl. Phys.* **30**(10), 1759–1761 (1959).

56. E. Galea and N. C. Markatos, The mathematical modelling and computer simulation of fire development in aircraft, *Int. J. Heat Mass Transfer* **34,** 181–197 (1991).
57. D. B. Spalding, Developments in the IPSA Procedure for Numerical Computation of Multiphase Flow Phenomena with Interphase Slip, Unequal Temperatures etc. *Numerical Properties and Methodologies in Heat Transfer* (Edited by T. M. Shih), pp. 421–436. Hemisphere, New York (1983).